Development of Complex Curricula for Molecular Bionics and Infobionics Programs within a consortial* framework**

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The Project has been realised with the support of the European Union and has been co-financed by the European Social Fund ***

**Molekuláris bionika és Infobionika Szakok tananyagának komplex fejlesztése konzorciumi keretben

***A projekt az Európai Unió támogatásával, az Európai Szociális Alap társfinanszírozásával valósul meg.

ELECTRICAL MEASUREMENTS

(Elektronikai alapmérések)

Theoretical approach to networks and systems

Hálózatok és rendszerek elméleti megközelítése

Dr. Oláh András

Outline

- Introduction to Circuit Theory
- Defnitions of corresponding quantities
- History of Circuit Theory
- Definition of elements
- The Kirchhoff laws
- Classificition of elements
- Linear resistive circuits
- Thevenin and Norton equivalent circuits
- System and Networks
- Linear dynamic circuits

Circuit theory

- **Motivation**: electric circuits are present almost everywhere, in home computers, television and hi-fi sets, electric power networks, telecommunication systems,etc. Circuits in these applications vary a great deal in nature and in the ways they are analyzed and designed.
- **Focus**: on the electrical behavior of circuits.
- **The goal**: it makes quantitative and qualitative predictions on the electrical behavior of circuits; consequently the tools of circuit theory will be mathematical, and the concepts and results pertaining to circuit will be expressed in terms of circuit equations and circuit variables, each with an obvious operational interpretation.

Quantities: charge, potential, voltage, current

- The concept of charge
  - The Coulomb [C] – the SI unit of charge. An electron carries -1.6e-19 [C]
  - Conservation of charge
- The concept of potential
  - Attraction/repulsion of charges
  - The electric field
  - The energy of moving a charge in a field
- Voltage is a difference in electric potential
  - always taken between two points (absolute voltage is a nonsensical fiction)
  - the concept of ground is also a (useful) fiction.
- It is a line integral of the force exerted by an electric field on a unit charge.
- Customarily represented by u or U. The SI unit is the Volt [V].

Quantities: current, power

Current is a movement of charge.

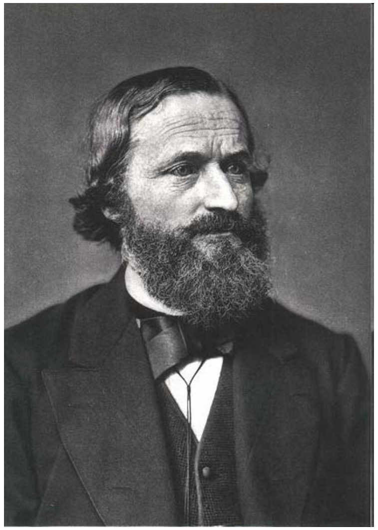
- It is the time derivative of charge passing through a circuit branch.
- Customarily represented by *i* or *I*.
- The SI unit is the Ampere [A].

Power is the product of voltage by current.

- It is the time derivative of energy delivered to or extracted from a circuit branch.
- Customarily represented by *p* or *P*. The SI unit is the Watt [W].

History of circuit theory

- Beginnigs:
  - early 1800s: Volta, Ampere, Ohm, Faraday, Henry, Siemens,
  - 1845: Kirchhoff's current and voltage laws
  - 1881: Maxwell
  - 1883: Thevenin
  - 1926: Norton
  - 1930: Bode
- What drives circuit theory?
  - Wired and wireless communications!
  - Computer technology.



Gustav Robert Kirchhoff
(1824 – 1887)

The elements: introduction

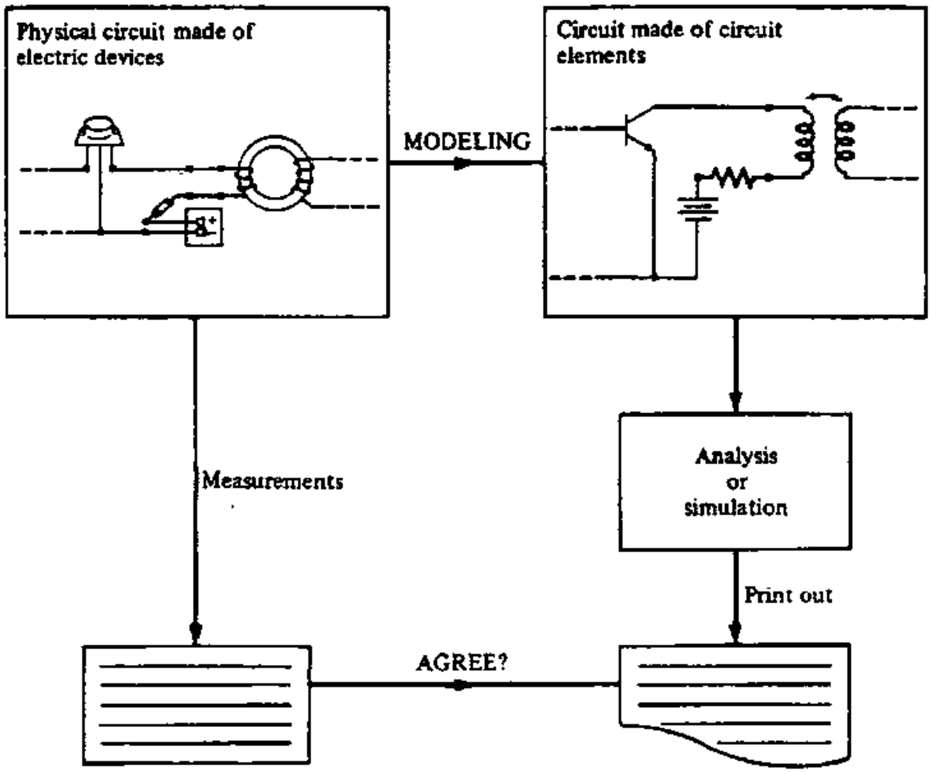
- A circuit is an assembly of elements whose terminals are connected at nodes (like a networks).
- There are basically seven kinds of elements that make up all circuits:
  - voltage source and current source
  - the resistor,
  - capacitor,
  - inductor,
  - diode and the transistor.
- We can construct circuits (or networks) as building blocks of such complex systems as computers, communication transceivers, audio-video entertainment systems, weapon systems, and medical diagnosis systems.

TÁMOP – 4.1.2-08/2/A/KMR-2009-0006

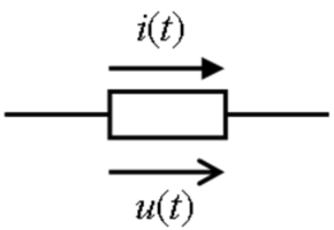
Electrice device vs. circuit elements

- By *electric device* mean the physical object in the laboratory or in the factory.
- Physical circuits are obtained by connecting electric devices by wires.
- We think of these electric devices in terms of *idealized models* named by *circuit elements* like:
  - the resistor ($u = Ri$),
  - the inductor ($u=L\ di/dt$),
  - the capacitor ($i = C\ du/dt$), etc.

Analyses of a physical circuit

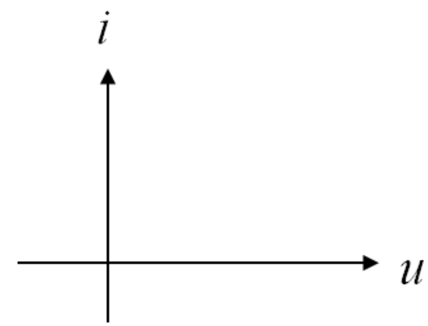


Characteristics of the elements



Implicite characteristics: $\Phi\left\{u(t), i(t)\right\} = 0$

Explicite characteristics:
$$u(t) = \Phi_u\left\{i(t)\right\}$$
$$i(t) = \Phi_i\left\{u(t)\right\}$$



The power: $p(t) = u(t)\cdot i(t)$ → $p(t) > 0$ → consumer; $p(t) < 0$ → producer

The work function: $w(t) = \int_{-\infty}^{t} p(\tau)\,d\tau$

The work between t_1 and t_2: $W(t_1, t_2) = \int_{t_1}^{t_2} p(t)\,dt = w(t_2) - w(t_1)$

Kirchhoff's laws

- The fundamental assumption of circuit theory is that the voltages satisfy Kirchhoff's voltage law (KVL):

$$u_1 + u_2 + \ldots + u_m = \sum_{k=1}^{m} u_k = 0$$

where the voltages form a loop,

- and the currents satisfy Kirchhoff's current law (KCL)

$$i_1 + i_2 + \ldots + i_n = \sum_{k=1}^{n} i_k = 0$$

where the currents meet at a node or are the terminal currents of an element.

Illustration to KVL

It can be illustrated by considering the situation of a mountain climber located half-way up a mountain. If that person walks around the mountain and returns to original starting point by any pathway and if all upward vertical distances are considered as positive and all downward vertical distance are considered as negative, the sum of all the vertical motion over the trip equals zero. Similarly, the sum of the voltage drops and rises around any closed loop equals zero.

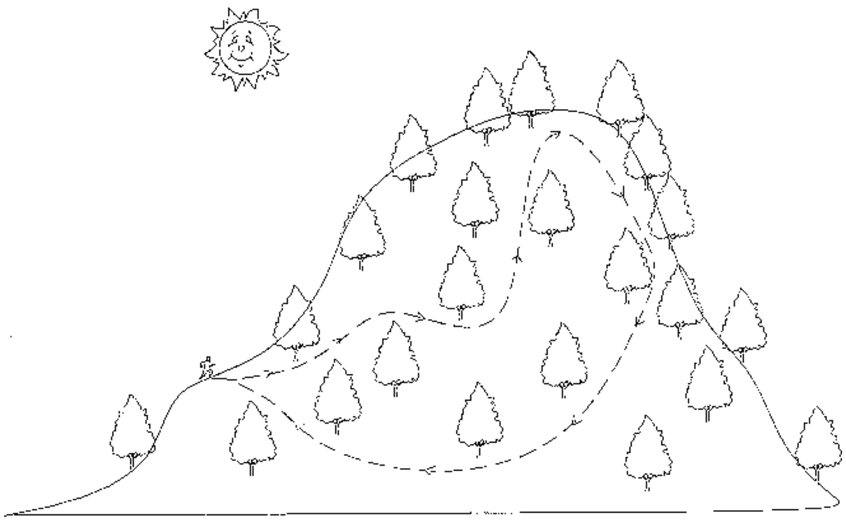
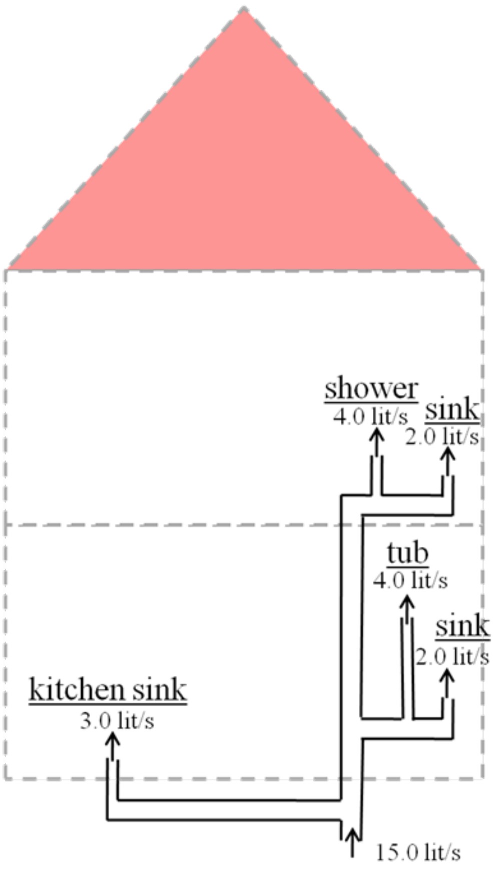


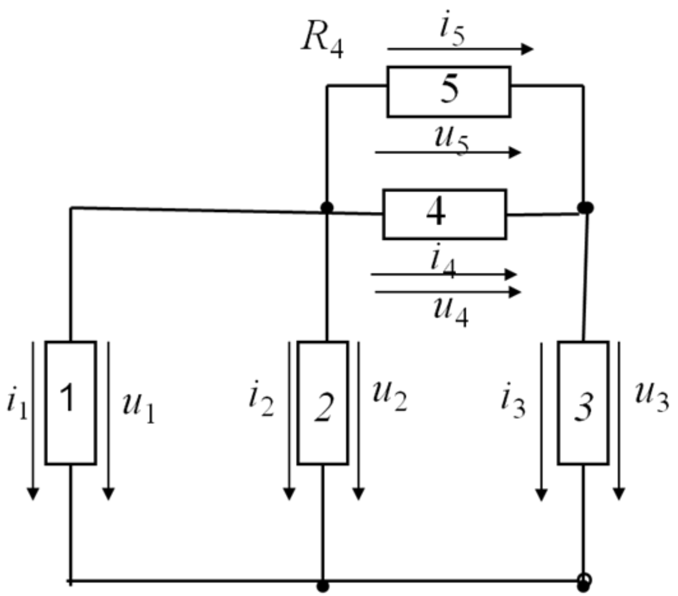
Illustration to KCL

Current can be represented as a liquid flow-moving across a fixed position in a unit of time. Thus, KCL can be illustrated by

- a series of **pipe connections in the home**, or
- a river bed that although the dimensions of a river may change from an upstream position to a downstream position, the mass flow across any lateral fixed position must be constant in liters per minute.



Linear homogeneous equations of circuit



KVLs:

$$u_2 - u_3 - u_4 = 0$$
$$u_1 - u_2 = 0$$
$$u_5 - u_4 = 0$$

KCLs:

$$i_1 + i_2 + i_4 + i_5 = 0$$
$$-i_5 - i_4 + i_3 = 0$$

~~$-i_1 - i_2 - i_3 = 0$~~

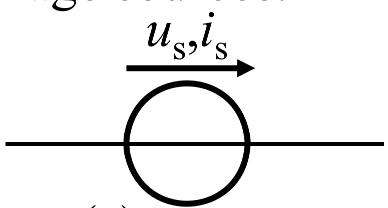
Linearly independent equations

Comment: 10 unknown quantities, 5 system equations and 5 characteristics

Solvable ⟶ Regular networks ⟶ Parametrical, Structural

Classification according to characteristics

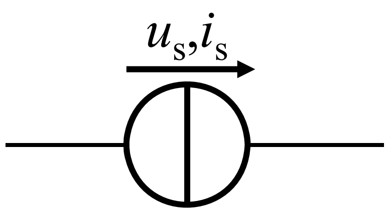
- Sources:
  - Voltage sources:



$u = u_s(t)$

i is arbitrary constraned by circuit

  - Current sources:



$i = i_s(t)$

u is arbitrary constraned by circuit

For sinusoidal signal: $U_{eff}=0.707U_{max}$

- Remarks:
  - DC: $u_s(t) = U_0$
  - AC: $u_s(t) = U_{max}\cos(\omega t + \varphi_0)$
  - Peak to peak value:
    $$U_{pp} = U_{max} - U_{min}$$
  - Effective value (RMS):
    $$U_{eff} = \sqrt{\frac{1}{T}\int_0^T u^2(t)\,dt}$$
  - Absolute value:
    $$U_{abs} = \frac{1}{T}\int_0^T |u(t)|\,dt$$
  - Average value: DC in AC
    $$U_0 = \frac{1}{T}\int_0^T u(t)\,dt$$

Classification according to characteristics

- **Resistive** (otherwise dynamic): $u(t=\tau)$ only depends on $i(t)$ at the time τ.

$$u(t=\tau)=\Phi_u\{i(t)\}=U(i(t=\tau),\tau) \qquad \forall\tau$$

$$\text{(or } i(t=\tau)=\Phi_i\{u(t)\}=I(u(t=\tau),\tau)\text{)}$$

- Example:
  - The resistor is resistive, because u is determined by i at time τ:

    $u=i\,R$(Ohm's „law”)
  - The inductor is dynamic, because $u(t=\tau)$ can not be determined by $i(t=\tau)$:

$$u_{\mathrm{L}}=L\frac{di_{\mathrm{L}}}{dt}$$

Classification according to characteristics

- **Linear** (otherwise nonlinear) elements fulfill the superposition principle:

$$\Phi_u\left(\sum_k C_k \cdot i_k(t)\right) = \sum_k C_k \cdot \Phi_u\left(i_k(t)\right) = \sum_k C_k \cdot u_k(t)$$

- Example:
  - The resistor is linear element (It's simple, show it!)
  - The inductor is linear, because

$$\Phi_u\{K_1 i_1 + K_2 i_2\} = L\frac{d\left(K_1 i_1 + K_2 i_2\right)}{dt} = K_1 L\frac{d\left(i_1\right)}{dt} + K_2 L\frac{d\left(i_2\right)}{dt} = K_1 \Phi_u\{i_1\} + K_2 \Phi_u\{i_2\}$$

Classification according to characteristics

- **Time invariant** (otherwise time variant) is one whose u voltage does not depend explicitly on time:

$$u(t)=\Phi_u\{i(t)\}\rightarrow u(t-\tau)=\Phi_u\{i(t-\tau)\}$$

- **Causal** (oterwise acausal) where the u voltage depends on past/current i current but not future inputs i.e. the $u(\tau)$ only depends on the $i(t)$ for values of $t<\tau$.

Classification according to characteristics

- **Passive** (otherwise active) elements consumes (but does not produce) energy, i.e the work function is always positive:

$$w(t) \geq 0.$$

- Example:
  - The resistor is passive element, because

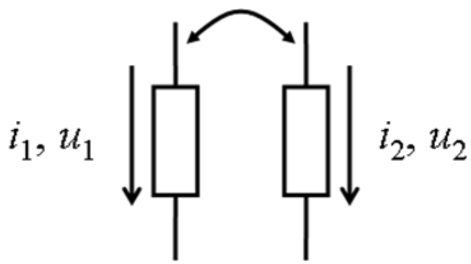
$$w(t) = \int_{-\infty}^{t} u \cdot i dt = \int_{-\infty}^{t} (Ri) \cdot i d\tau = R\int_{0}^{t} i^2 dx \geq 0$$

only if R>0 !

  - The inductor is passive, because

$$w(t) = \int_{-\infty}^{t} u \cdot i dt = \int_{-\infty}^{t} L\frac{di}{d\tau} \cdot i d\tau = \frac{L}{2}\int_{0}^{i^2} dx = \frac{L}{2} i^2(t) \geq 0$$

Coupled two-terminal elements



$$\left.\begin{aligned}\Phi_1\{u_1,u_2,i_1,i_2\}=0\\ \Phi_2\{u_1,u_2,i_1,i_2\}=0\end{aligned}\right\}$$ Implicite characteristics

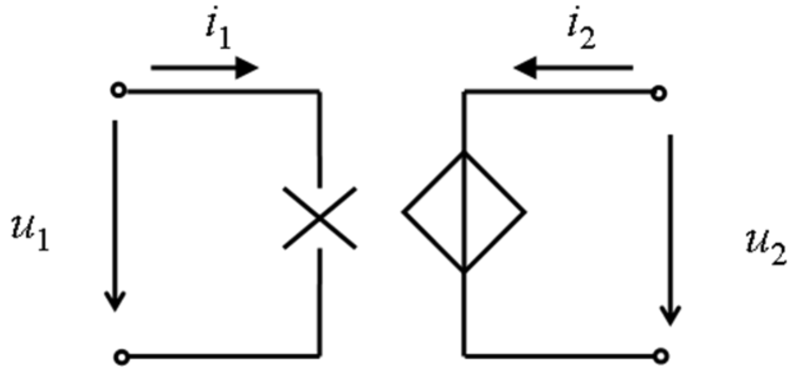
$$\left.\begin{aligned}
i_1 &= \Phi_{i_1}\{u_1,u_2\} & u_1 &= \Phi_{u_1}\{i_1,i_2\}\\
i_2 &= \Phi_{i_2}\{u_1,u_2\} & u_2 &= \Phi_{u_2}\{i_1,i_2\}\\
i_1 &= \Phi_{i_1}\{u_1,i_2\} & u_1 &= \Phi_{u_1}\{i_1,u_2\}\\
u_2 &= \Phi_{u_2}\{u_1,i_2\} & i_2 &= \Phi_{i_2}\{i_1,u_2\}
\end{aligned}\right\}$$ Explicite characteristics

Examples:

- Transformator
- Controlled sources
- Girator
- …

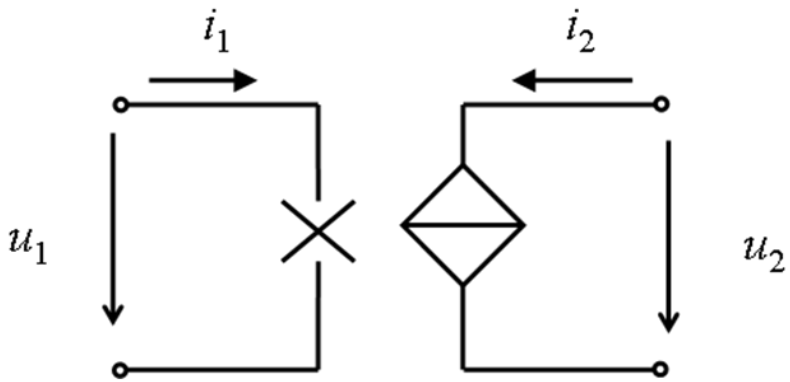
Voltage controlled sources

Voltage controlled voltage source



$$i_1 = 0$$
$$u_2 = \mu \cdot u_1$$

Voltage controlled current source

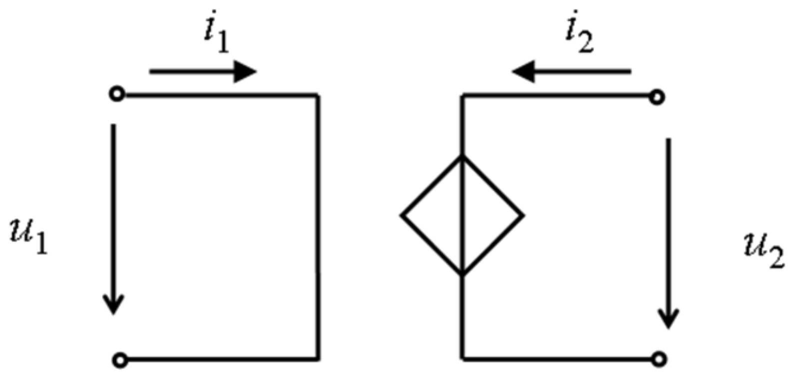


$$i_1 = 0$$
$$i_2 = g \cdot u_1$$

Field Effect Transistor
[→see Chapter 7.]

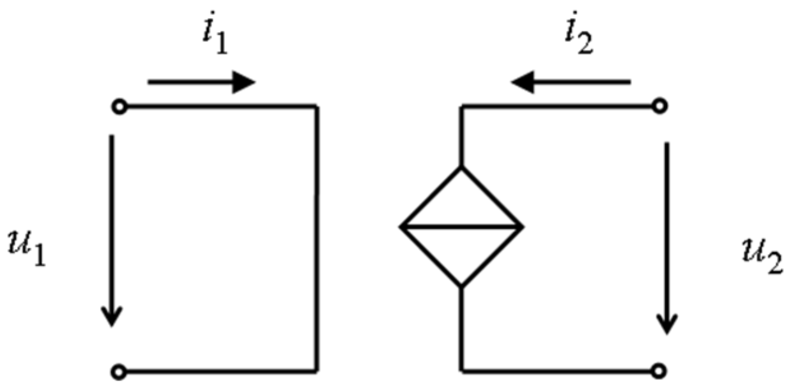
Current controlled sources

Current controlled voltage source



$$u_1 = 0$$
$$u_2 = \beta \cdot i_1$$

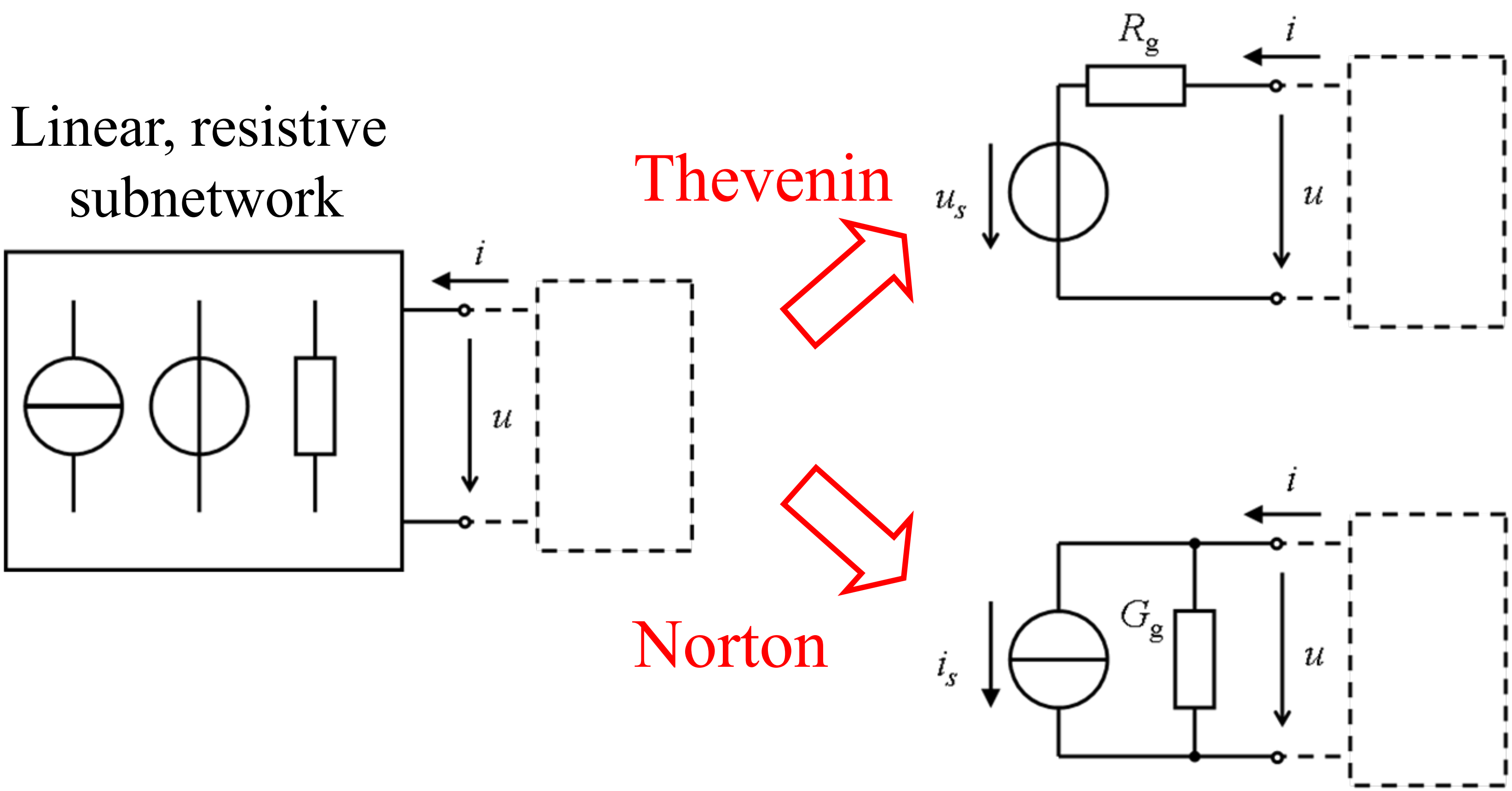
Current controlled current source



$$u_1 = 0$$
$$i_2 = \alpha \cdot i_1$$

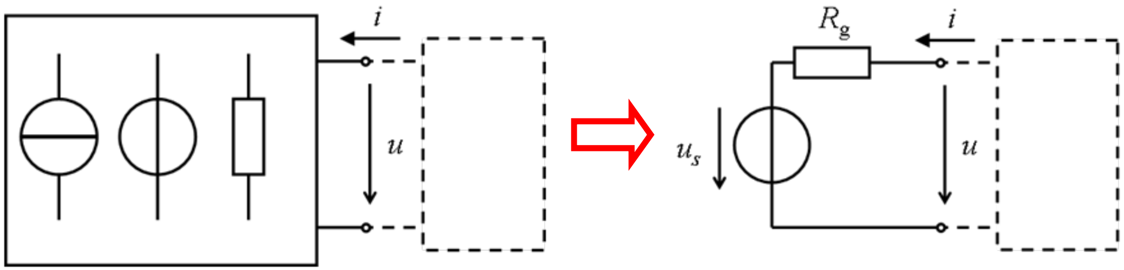
Bipolare Transistor
[→see Chapter 7.]

Thevenin and Norton equivalent circuits



Thevenin equivalent circuits

- **Problem**: Find the Thevenin equivalent voltage at the output.
- **Solution:**
  - **Known Information and Given Data**: Circuit topology and values.
  - **Unknowns**: Thevenin equivalent voltage u_s.
  - **Approach**: Voltage source u_s is defined as the output voltage with no load.
  - **Assumptions**: None.



Léon Charles Thévenin
(1857–1926)

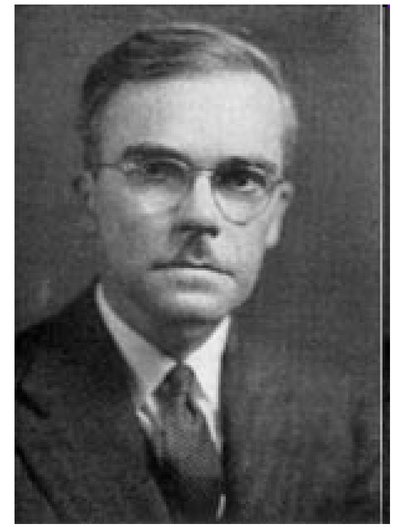
http://en.wikipedia.org/wiki/File:Leon_Charles_Thevenin.jpg

Thevenin equivalent circuits (cont')

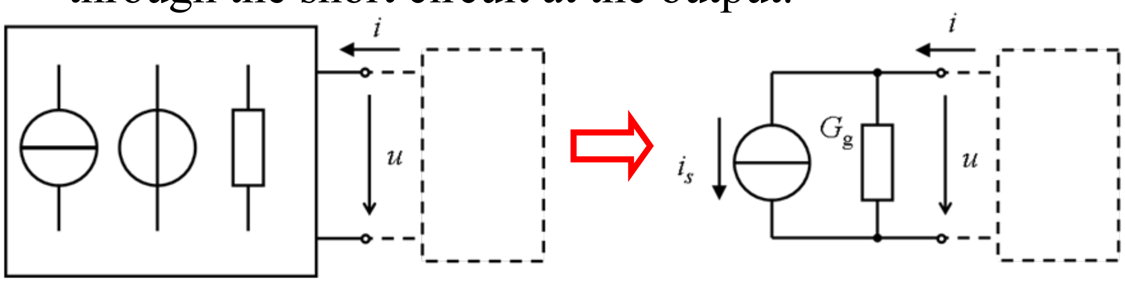
- **Problem**: Find the Thevenin equivalent resistance.
- **Solution:**
  - **Known Information and Given Data**: Circuit topology and values.
  - **Unknowns**: Thevenin equivalent voltage u_s.
  - **Approach**: When zeroing a current source, it becomes an open circuit. When zeroing a voltage source, it becomes a short circuit.
  - We can find the Thevenin resistance by zeroing the sources in the original network and then computing the resistance between the terminals.
  - **Assumptions**: None.

Norton equivalent circuits

- **Problem**: Find the Norton equivalent current at the output.
- **Solution:**
  - **Known Information and Given Data**: Circuit topology and values.
  - **Unknowns**: Norton equivalent short circuit current i_s.
  - **Approach**: Evaluate current through output short circuit. A short circuit has been applied across the output. The Norton current is the current flowing through the short circuit at the output.



Edward Lawry Norton
(1898–1983)

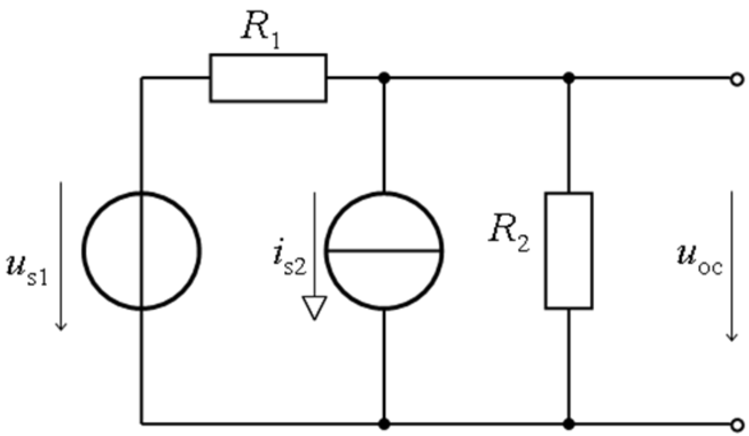


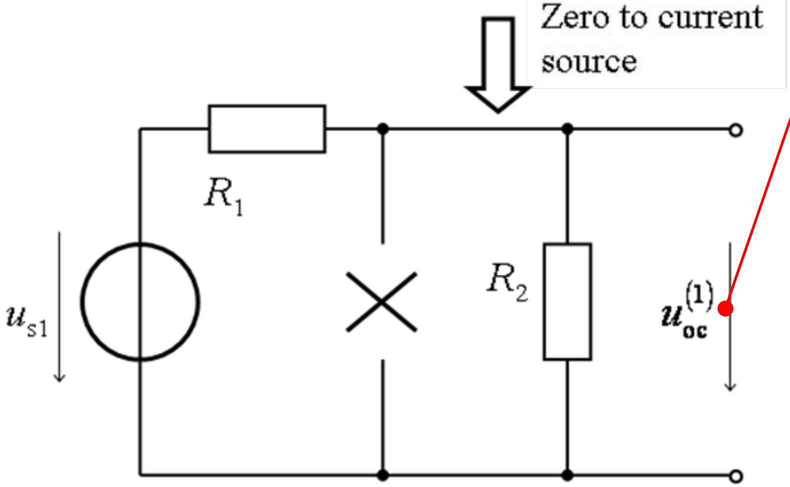
http://en.wikipedia.org/wiki/File:Nortonscientist.jpg

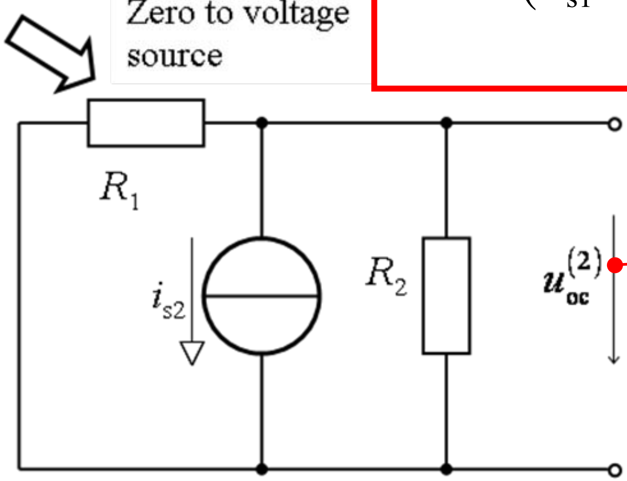
Step-by-step in Thevenin / Norton equivalent

- Perform three of these:
  - Determine the open-circuit voltage $u_s = u_{oc}$.
  - Determine the short-circuit current $i_s = i_{sc}$.
  - Zero the sources and find the Thévenin resistance R_g looking back into the terminals.
- Use the equation $u_s = R_g i_s$ to compute the remaining value.
- The Thevenin equivalent consists of a voltage source u_s in series with R_g .
- The Norton equivalent consists of a current source i_s in parallel with R_g.

Example







superposition principle

$$u_{oc}^{(1)} = u_{s1}\frac{R_2}{R_1 + R_2}$$

$$u_{oc}^{(2)} = -i_{s2} \cdot R_1 \otimes R_2$$

$$u_s = u_{oc}^{(1)} + u_{oc}^{(2)} = (u_{s1} - i_{s2}R_1)\frac{R_2}{R_1 + R_2}$$

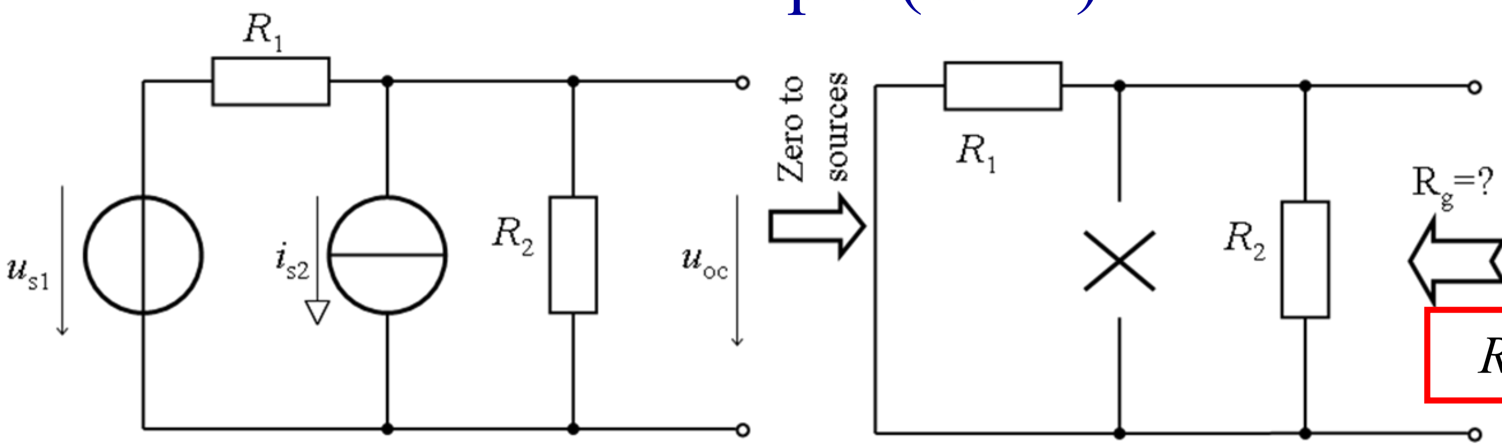
Superposition principle

- The superposition principle states that the total response is the sum of the responses to each of the independent sources acting individually. This method is only applicable to linear systems.
- In equation form, this is

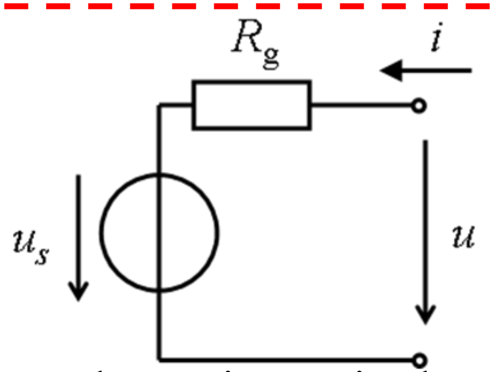
$$u = u^{(1)} + u^{(2)} + u^{(3)} + \cdots + u^{(n)}$$

where now n is the number of sources.

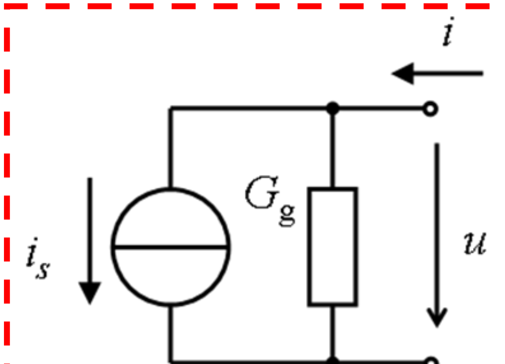
Example (cont')



$$R_g = R_1 \otimes R_2$$



Thevenin eguivalent

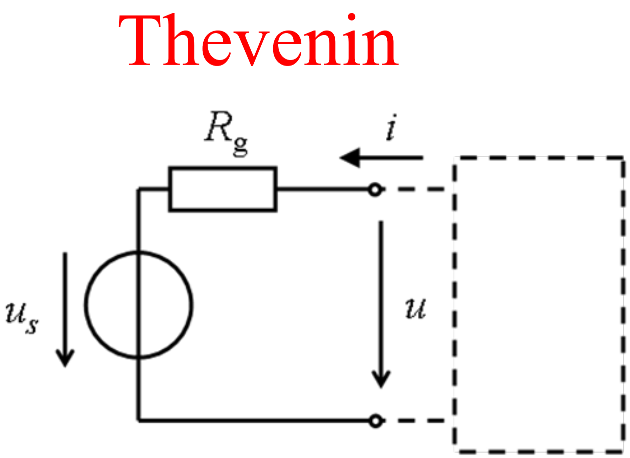


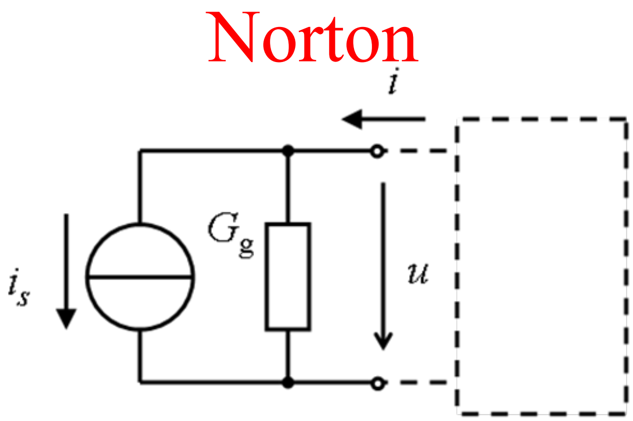
$$G_g = 1 / R_g$$

$$i_s = -\frac{u_s}{R_g}$$

Norton eguivalent

Thevenin vs. Norton equivalent circuits

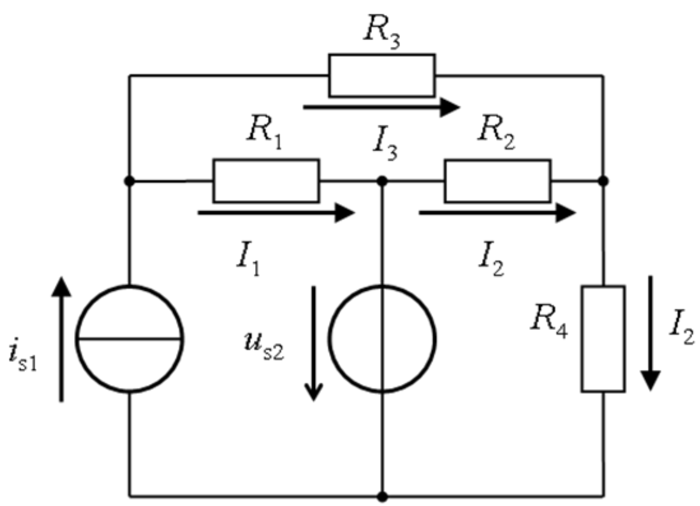




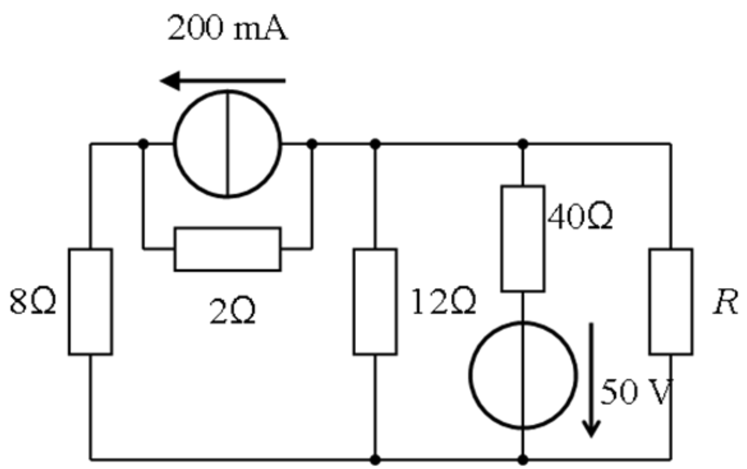
While the two circuits are identical in terms of voltages and currents at the output terminals, there is one difference between the two circuits. With no load connected, the Norton circuit still dissipates power!

Applications: source transformations

Example 1

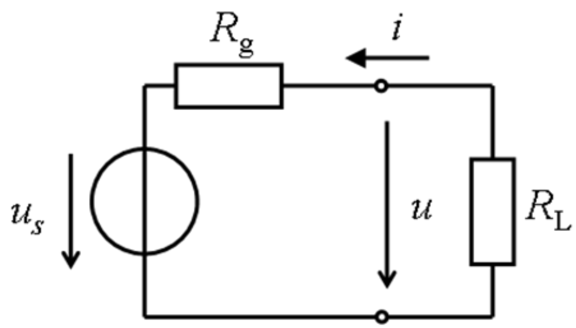


Example 2



Applications: maximum power transfer

- **Statement**: The load resistance (R_L) that absorbs the maximum power from a two-terminal circuit is equal to the Thevenin resistance (R_g).
- **Proof:**



$$R_\mathrm{L}^{(\mathrm{opt})} : \max_{R_\mathrm{L} \geq 0} P_{R_\mathrm{L}}$$

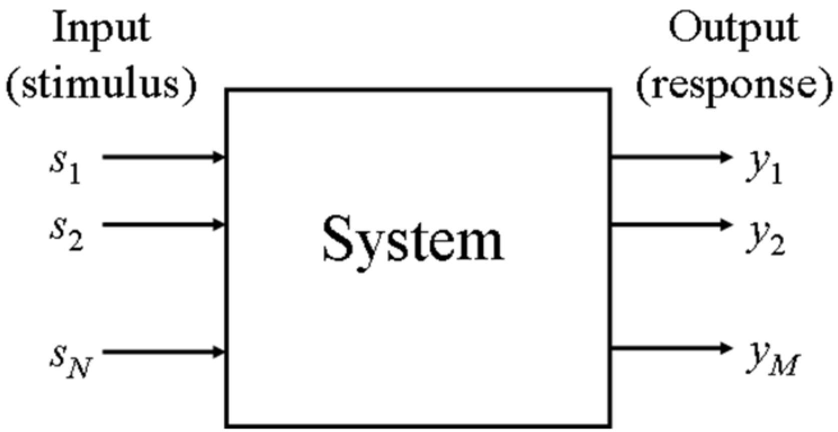
$$P_{R_\mathrm{L}} = R_\mathrm{L} \cdot i^2 = R_\mathrm{L} \left(\frac{u_s}{R_\mathrm{L} + R_\mathrm{g}} \right)^2$$

$$\frac{dP_{R_\mathrm{L}}}{dR_\mathrm{L}} = \ldots = \frac{R_\mathrm{g} - R_\mathrm{L}}{\left(R_\mathrm{g} + R_\mathrm{L}\right)^2} u_\mathrm{s}^2 \stackrel{!}{=} 0$$

$$\boxed{R_\mathrm{L}^{(\mathrm{opt})} = R_\mathrm{g}} \longrightarrow P_{R_\mathrm{L}}^{(\mathrm{opt})} = \frac{u_\mathrm{s}^2}{4R_\mathrm{g}}$$

System vs. Networks (or circuits)

- The system is the model of a physical object, the network is one of its realizations (implementations)
- Tasks:
  - Networks (or circuit) analyses (corresponding system identification)
  - Networks synthesis (system implementation)

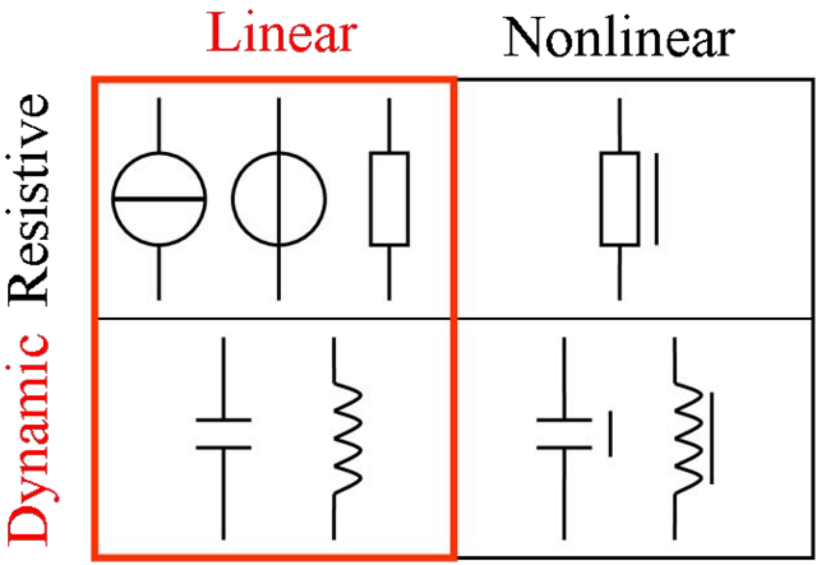


The system function

$$\mathbf{y} = \Psi(\mathbf{x})$$

Linear dynamic circuits (or networks)

- Basic linear elements:
  - Resistor, R, [Ω] (Ohms)
  - Inductor, L, [H] (Henrys)
  - Capacitor, C, [F] (Farads)



Inductors

- Current in an inductor generates a magnetic field:

$$B(t) = K_1\, i_L(t)$$

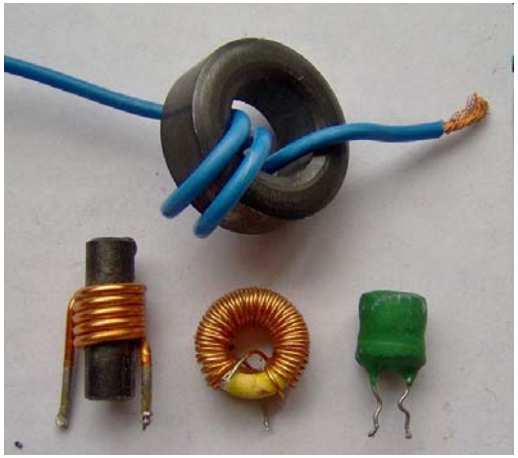
- Changes in the field induce an inductive voltage:

$$u_L(t) = K_2 \frac{dB(t)}{dt}$$

- The instantaneous voltage is

$$u_L(t) = \Phi_u\{i_L(t)\} = L\frac{di_L(t)}{dt}$$

where $L = K_1 K_2$.



http://en.wikipedia.org/wiki/File: Electronic_component_inductors.jpg

Capacitors

- Charge in a capacitor produces an electric field E, and thus a proportional voltage,

$$Q = C\, u_C(t),$$

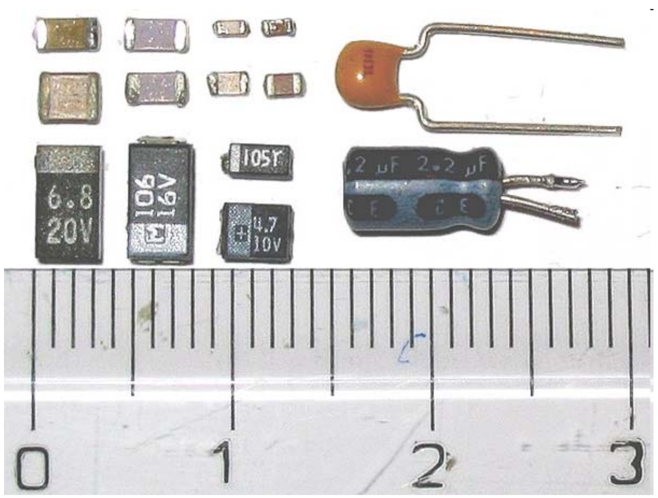
where C is the capacitance.

- The charge on the capacitor changes according to

$$i_C = \mathrm{d}Q/\mathrm{d}t.$$

- The instantaneous current is therefore

$$i_C(t) = \Phi_i\{u_C(t)\} = C\frac{du_C(t)}{dt}$$



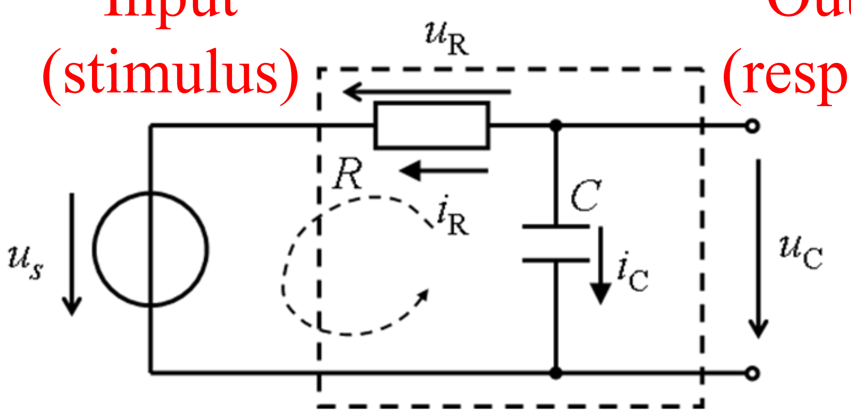
http://en.wikipedia.org/wiki/File:Photo-SMDcapacitors.jpg

Linear dynamic circuits (or networks)

- Example:

Input (stimulus)

Output (response)



System (lowpass filter)

KVL and KVC:

$$u_s - u_C + u_R = 0$$

$$i_R + i_C = 0$$

Charateristics of elements:

$$i_C = C\frac{du_C}{dt}$$

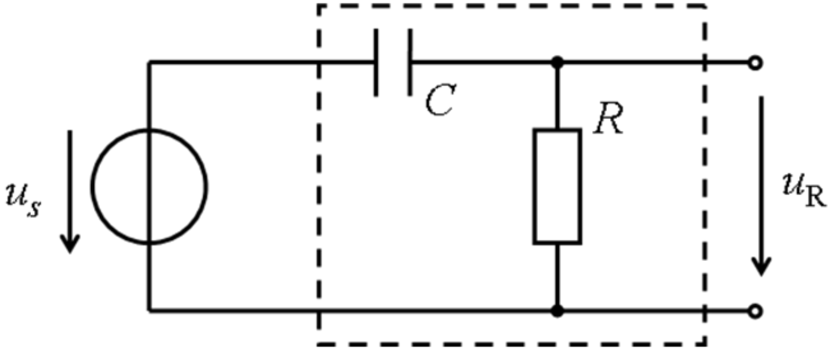
$$u_R = R \cdot i_R$$

$$u_s = u_s(t)$$

The system function:
first-order linear ordinary differential equation

$$RC\frac{du_C(t)}{dt} - u_C(t) + u_s = 0$$

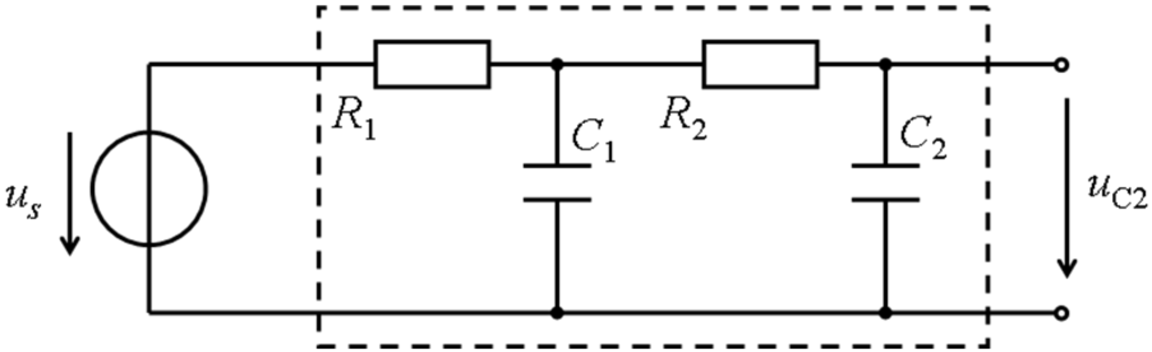
Problems: Give the system functions!



System (high pass filter)

First-order linear ordinary differential equation

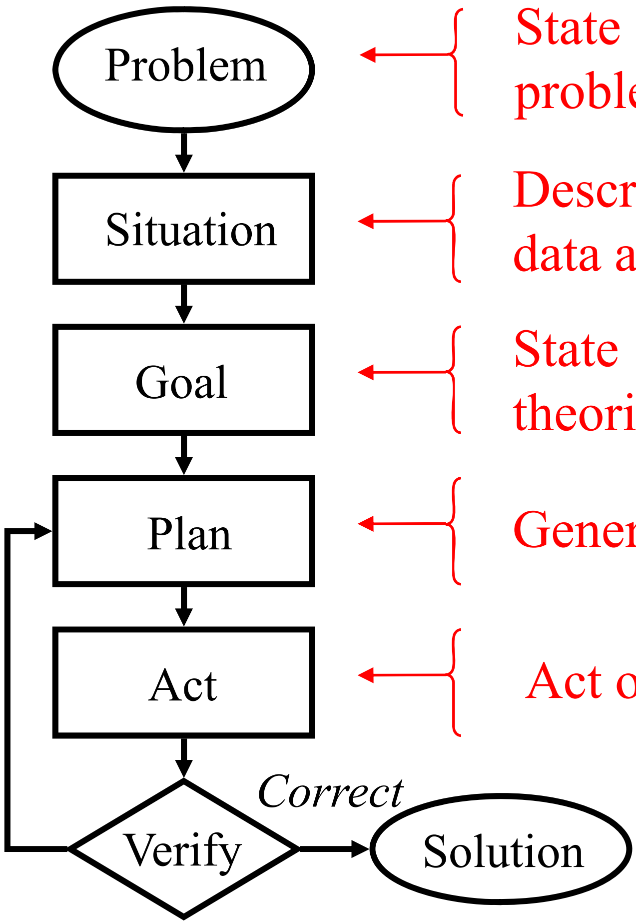
You can able to solve it after Analysis Course



System (lowpass filter)

Second-order linear ordinary differential equation

Problem-Solving Approach for measuerments in the lab



State the problem: recognize and understand the problem.

Describe the situation and assumptions. Gather data and verify its accuraccy

State the goals and requirements select guiding theories and principles (eg.: Norton equivalent)

Generate a plan to obtain a solution of the problem

Act on the plan

Verify that the proposed solution is indeed correct and present the solution

Summary

- Circiut theory makes quantitative and qualitative predictions on the electrical behavior of circuits.
- A circuit is an assembly of elements whose terminals are connected at nodes (like a networks)
- The system is the model of a physical object, the network is one of its realizations (implementations).
- The system is fully characterized by system function (eg.:transfer funtion).
- Linear resistive circuits consists linear resistive (and time invariant) elements.
- For linear resistive circuits there are equivalent circuits consisting a resistor and a current (Norton equivalent) or voltage (Tevenin equivalent) source.
- Linear dynamic circuits consists linear resistive and dynamic elements (which are all time invariant).

- ***Next lecture:*** *Semiconductors basics: diodes and transistors*