

ECONOMIC STATISTICS





NEW

SZÉCHENYI PLAN

ECONOMIC STATISTICS

Sponsored by a Grant TÁMOP-4.1.2-08/2/A/KMR-2009-0041

Course Material Developed by Department of Economics,

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The project is supported
by the European Union.

National Development Agency
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The projects have been supported
by the European Union.

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ECONOMIC STATISTICS

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June 2010

ECONOMIC STATISTICS

Week 4

Simple regression – fit,
nonlinearity, confidence interval

Anikó Bíró

Simple regression – reminder

- Regression model:

$$Y_i = \alpha + \beta X_i + e_i$$

$$Y_i = \hat{\alpha} + \hat{\beta} X_i + u_i$$

- Estimation: OLS

Example

70 tropical countries, relationship between X:
population density (capita/1000 ha), and Y:
deforestation rate (%)

	<i>Coefficients</i>
Intercept	0,60
X variable	0,001

Interpretation?

Measure of fit

- OLS: finding the best fitting line
- How good is the fit?
 - Measure: R^2
 - Simple (univariate) regression:
square of correlation = R^2

Estimated value

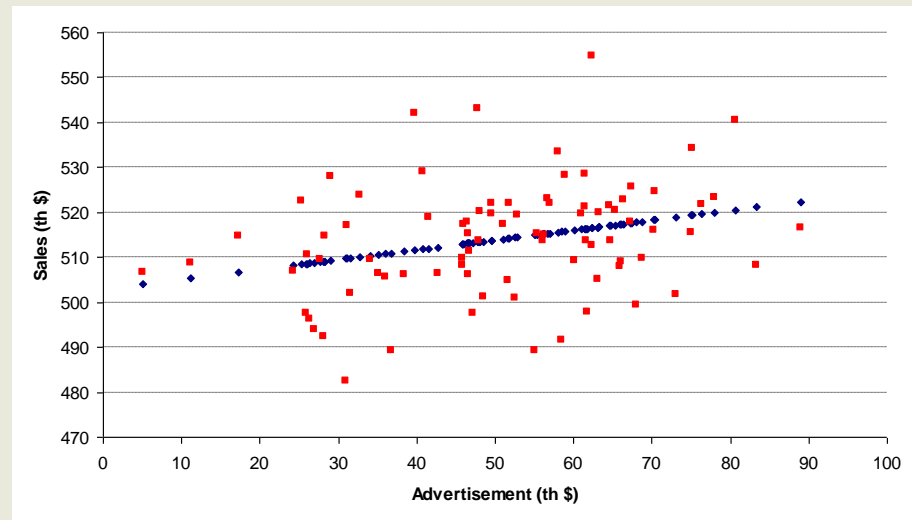
- Regression line: $Y = \alpha + \beta X + e$
- Estimated/fitted/forecasted value: $\hat{Y} = \hat{\alpha} + \hat{\beta}X$

Comparison of the two – how good is the fit

Advertisement example:

Residual:

$$u = Y - \hat{Y}$$



Residual vs. error term!

R^2

Total sum of squares : $TSS = \sum (Y_i - \bar{Y})^2$

Variance : $TSS/(N - 1)$

Regression sum of squares : $RSS = \sum (\hat{Y}_i - \bar{Y})^2$

Sum of squared residuals : $SSR = \sum (Y_i - \hat{Y}_i)^2 = \sum u_i^2$

$$TSS = RSS + SSR$$

$$R^2 = 1 - \frac{SSR}{TSS} = \frac{RSS}{TSS}$$

Interpret R^2

- What percentage of the variance of Y is explained by X

$$0 \leq R^2 \leq 1$$

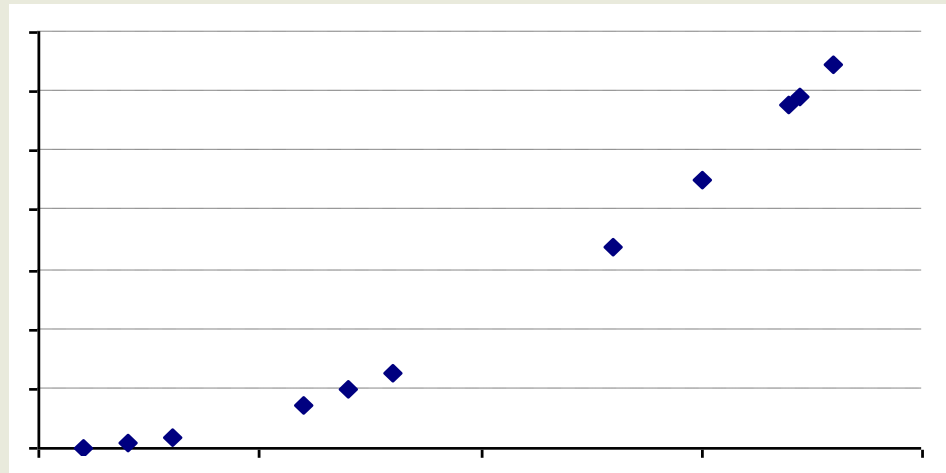
- $R^2=1$ – perfect fit

Nonlinearity

Nonlinear relationship between X
and Y

Common examples:

- Quadratic:



- Logarithmic

Logarithmic form

- Can ensure linear relationship
- Easy to interpret – elasticity:

$$\ln Y = \alpha + \beta \ln X$$

$$\beta = \frac{d \ln Y}{d \ln X}$$

If X increases by one %, Y increases by beta % on average

- Unit of measurement does not matter
- Approximation of % change: $100 \cdot d \ln Y$

Logarithmic form, cont.

How to interpret the slope coefficients?

$$Y_i = \alpha + \beta \ln X_i + e_i$$

$$\ln Y_i = \alpha + \beta X_i + e_i$$

Uncertainty

- Real values of the coefficients are unknown
- Estimated based on a sample
 - Estimated value is not exactly equal to the true value
- Point estimation: does not reveal the uncertainty

Factors influencing the precision of the OLS estimation

See textbook graphs

- More observations – more precise estimation
- Smaller error terms – more precise estimation
- Larger variance of X – more precise estimation
 - Example: effect of education level on income

Confidence interval

$$\left(\hat{\beta} - t_b s_b, \hat{\beta} + t_b s_b \right)$$

$$s_b = \sqrt{\frac{SSR}{(N-2) \sum (X_i - \bar{X})^2}}$$

s_b : standard deviation of $\hat{\beta}$

t_b : Student's t - distribution

Larger confidence level \rightarrow larger t_b

More observations \rightarrow smaller t_b

Interpretation

- Most common:
 - 95% confidence interval
 - ”There is 95% chance that the true value of the coefficient lies in the given interval”
- Large N, 95%: $t=1,96$
- Table of t-distribution
- Excel: confidence level can be chosen

Deforestation example

	Coefficients	<i>Standard dev.</i>	<i>Bottom 95%</i>	<i>Top 95%</i>
Intercept	0,6000	0,1123	0,3758	0,8241
X variable	0,0008	0,0001	0,0006	0,0011

Summary

- Interpretation of estimated coefficients
- R-squared
- Nonlinearity, logarithmic form
- Uncertainty, confidence interval

Simple regression – fit, nonlinearity, confidence interval

Seminar 4

R^2

Total sum of squares : $TSS = \sum (Y_i - \bar{Y})^2$

Variance : $TSS/(N - 1)$

Regression sum of squares : $RSS = \sum (\hat{Y}_i - \bar{Y})^2$

Sum of squared residuals : $SSR = \sum (Y_i - \hat{Y}_i)^2 = \sum u_i^2$

$$TSS = RSS + SSR$$

$$R^2 = 1 - \frac{SSR}{TSS} = \frac{RSS}{TSS}$$

Interpret R^2

- What percentage of the variance of Y is explained by X

$$0 \leq R^2 \leq 1$$

- $R^2=1$ – perfect fit
- Examples: advertisement regression, KSH unemployment rate regression

Examples for nonlinearity

Textbook: 4.5, 4.6

Uncertainty

- Real values of the coefficients are unknown
- Estimated based on a sample
 - Estimated value is not exactly equal to the true value
- Point estimation: does not reveal the uncertainty
- Confidence interval:

$$\left(\hat{\beta} - t_b s_b, \hat{\beta} + t_b s_b \right)$$
$$s_b = \sqrt{\frac{SSR}{(N-2) \sum (X_i - \bar{X})^2}}$$

Examples

- Advertisement – sales example:
confidence interval of the estimated
slope parameter (various confidence
levels)
- Real estate prices – lot size
(hprice.xls)

Homework 3 (groups)

- Analyze the relationship between two variables from a cross sectional sample (KSH, Eurostat, OECD, Penn World tables)
 - Descriptive statistics of both variables
 - Correlation
 - Regression
 - Functional form?
 - Fit?
 - Interpret the estimation results (confidence interval, as well)